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# DETERMINING OF STRESS-STRAIN STATE OF THE CASING STRING ACCORDING TO THE DIRECTIONAL SURVEY DATA DURING THE WELLBORE CONSTRUCTION

***Abstract:*** *The casing string in the curvilinear borehole is represented as a long elastic rod, for which a non-uniform system of differential equations is constructed and integrated taking into account its own weight and friction. Formulas for the distribution of axial forces and bending moments in the body of the column, as well as the reactions of the walls leading the column to the actual well profile are obtained. To calculate these force factors, a method for numerical integration of inclinometric measurements data and software for numerical analysis of a real well are developed. This technique allows to detect the areas of local increase of the curvature and difficult passage of the curvilinear well and calculate the parameters of the stress-strain state of the casing column in it.*

***Keywords:*** *casing string, curved well, inclinometric measurements, axial force, wall reaction.*

## Introduction

The technology of reliable and safe extraction of oil and gas from large depths requires the borehole wall lining by a string of casing. Modern methods of directional and horizontal drilling allow to reach productive layers at a depth of 4÷6 km with a string length of 5÷8 km, while steel pipes have a diameter of only 168÷140 mm with a wall thickness of 10÷12 mm. The main production casing, which connects the wellhead with deposits of hydrocarbons, must be continuous, strong and pressure-tight.

A typical well program includes a vertical section, one or more inclined sections (which provide a large deviation from the vertical one) and a vertical bottom-hole section. The straight-line areas are interconnected by transition curved ones, which are described by the circle arc of a constant radius. When designing, all wells are usually located in one vertical plane.

While drilling there are deviations from the well design profile, which are continuously corrected by technical and technological means. As a result, the area of the real well is not exactly straight or circular arc, but contains local distortions and deviations from the given shape. To establish the real profile of the drilled well, its logging is conducted, during which zenith angle $\vartheta$ between the tangent to the curved axis of the well and the vertical is measured. According to the data obtained, an inclinometric table is compiled demonstrating the table dependence of the angle $\vartheta(s)$ from the coordinate $s$, which is the distance from the earth surface along the curved axis of the well to the specified intersection. Measurements are carried out from the wellhead to the bottom with a certain pitch $\Delta s$.

The production casing, lowered into the curved well, enters into force interaction with the wells due to the rigidity of the pipes. Due to the reaction of the walls, the string bends, resembling the profile of the well. As a result, in the pipes body there emerges a complex stress-strain state caused by their bending and axial tension, which greatly affects the reliability and durability of the casing.

## Literature review

The work of Yu. Pesliak [1] is devoted to the study of internal stresses in the columns of oil wells. The problem of determining the forces acting on the string of pipes in the well when its shape is given is considered here. To solve it, a system of G. Kirchhoff equations describing the spatial deviations of a long elastic rod, which has a finite bending stiffness, has been used, and its solution in a vector form has been carried out. However, the results in scalar form suitable for engineering calculations are obtained only for the case of a well section, which is curved along a circular arc of a constant radius in one plane, and for a well case, which is presented by a helix with a constant zenith angle and a constant rate of change of the azimuthal angle. In order to determine the axial forces and frictional forces in the case of random deviation of the well, numerical integration is applied on an example of a well with a constant speed of change of the zenith and azimuthal angle to their maximum value of 90°.

In the paper by P. Vyslobitskyi [2] the problem of the advance and bending of the string of pipes in the deviated borehole is considered. For its solution a geometric approach is used to study the force interaction of pipes with well walls. At the same time, the pipe was graphically inscribed into well deviated along the circular arc following several possible, according to the author, schemes of placement of contact points with its walls, in which reactions and frictional forces may occur. The acting forces were determined by the equilibrium equations of the pipe sections between these contact points and the pipe deformation equations, for which the formulas of the small deformations of the cantilever beam were unjustifiably applied. With the general formulation of the problem of bending and casing string drift in the deviated wellbore, a system of differential equations was proposed. The system had to express the bent state of the string, but it does not contain two equations of equilibrium of internal and external forces projections, and therefore it is incomplete and cannot be solved.

Thus, *unresolved remains the problem* of obtaining a closed system of differential equations, which describes the deformation of the string of pipes in a deviated well under their own weight and the reactions of the walls. The solutions of this problem will determine the distribution of axial forces, bending moments and stresses in the body of the column.

## Purpose

A long casing string in the well behaves like an elastic, solid rod [1, 3], which has sufficient bending stiffness. It is influenced by a vertical weight $j$, uniformly distributed along the length, which creates variable axial forces in the body of the column. The column of initially rectilinear pipes in the curvilinear well forcibly receives the geometric shape of its curved axis. This is due to the reaction forces of the well walls, which, together with the weight, act on the column and bend it.

In the first approximation, we consider that the column contacts the well walls along its entire length (we neglect small gaps between the wall and the pipe in comparison with large geometric deviations of the axis from the rectilinear form). Consequently, along with the distributed weight $j$, a long elastic rod is influenced by the reaction of the walls $f(s)$, distributed by its length according to a certain law, as a result of which it acquires a given form. We assume that the distributed load $f(s)$ is directed along the normal to the curvilinear axis of the rod and is positive if its projection to the horizontal has a positive direction.

*The purpose of the work* is to develop a method for determining the distribution of axial forces and the reactions of the walls, which, together with their own weight, act on a casing string and make it follow a given wellbore shape. To do this, it is necessary to derive and integrate the system of differential equations of equilibrium of a long elastic rod bent in one plane. According to the obtained results, it is necessary to find expressions of force parameters that describe the stress-deformed state of the casing in a deviated well.

## The basic system of differential equations

The analysis has showed that large elastic deformations of the long rod of the unit-value stiffness can be considered for the bend, without losing the universality of the solution [3]. At the same time, the bending moment is numerically equal to the curvature of the rod, and the current force factors differ in size from the estimated ones by the factor *EJ* (*E* is the elastic modulus of the material, *J* is the moment of inertia of the cross-section of the rod).

Let us consider the arc element – the segment of the curved axis of the rod with length *ds*, at the beginning of which the tangent line is inclined to the vertical under zenith angle $\vartheta$ (fig. 1). In this section, the internal axial *t* and transverse *u* forces as well as bending moment *q* are applied.

In the final crosscut of the element, which received an increase in zenith angle $d\vartheta$, the same forces are applied, but with increments *dt*, *du*, *dq* correspondingly, the direction of which must balance the initial ones. The element is also affected by external forces: its weight *j ds*, the reaction of the wall *f ds* and the friction force $k_t f\,ds$, directed along the axis of the string against its motion (not shown in fig. 1), where $k_t$ is the coefficient of friction. In the equations of equilibrium, discussed below, the sign of friction corresponds to the descent of the string in the well. For the case of lifting a column in the equations and their solutions, the coefficient of friction should be taken with the opposite sign.

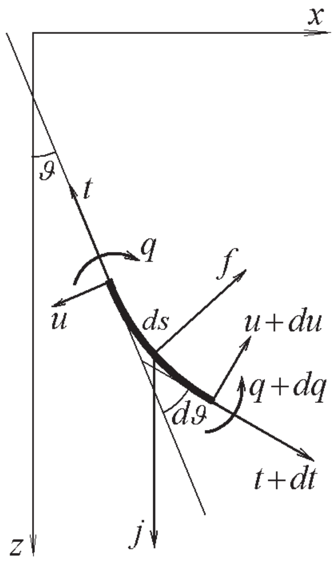


***Figure 1.*** *Calculation schema of a curved elastic element*

Let us project all forces on a normal and tangent line, make an equation of equilibrium of their projections, as well as equations of equilibrium of finite moments and moments of finite forces, and obtain a system of differential equations:

$$\frac{du}{ds}+t\cdot\frac{d\vartheta}{ds}-j\sin\vartheta+f=0$$

$$\frac{dt}{ds}-u\cdot\frac{d\vartheta}{ds}+j\cos\vartheta-k_t f=0$$

$$\frac{dq}{ds}+u=0$$

A similar system describing the bending deformation of a long elastic rod in one plane was obtained in the work of R. Frisch-Fay [4]. However, it did not take into account the reaction of the wall and friction, and was incomplete. In order for the system to have a solution, the fourth equation is required, which is the kinematic Euler equation. It establishes the connection between the angular deformation of the rod and its curvature $q$ (bending moment)

$$q=\vartheta'=\frac{d\vartheta}{ds}=\frac{1}{R}$$

where $R$ is local radius of curvature; the dot denotes the derivative of $s$.

Due to this, the system of differential equations becomes closed and has a solution.

Thus, the deformations of casing string, bent due to the deviation of the borehole, are described by a non-uniform system of four differential equations:

$$u'+t\cdot\vartheta'=j\sin\vartheta-f \tag{1}$$

$$t'-u\cdot\vartheta'=-j\cos\vartheta+k_t f \tag{2}$$

$$q'+u=0 \tag{3}$$

$$\vartheta'-q=0 \tag{4}$$

As we see, this system (1)-(4) contains three unknown functions $t$, $u$ and $q$ (which are internal force factors) and an unknown function of the distributed reaction $f$ (which is an external load). The function $\vartheta$ is known due to the inclinometric table of well measurement. It is necessary to solve the inverse problem – having the known load $j$ and the deformations $\vartheta$ given by the shape of the well, it is necessary to determine the unknown internal forces $t, u, q$ and such an external load function $f$, which creates a given shape of the rod.

Applying the equation (3)-(4), we reduce the system to two equations:

$$t\cdot\vartheta'-\vartheta'''=j\sin\vartheta-f \tag{5}$$

$$t'+\vartheta''\cdot\vartheta'=-j\cos\vartheta+k_T f \tag{6}$$

The system of differential equations (5)-(6), where the zenith angle function $\vartheta$ is known, contains two unknown functions $t$ and $f$. According to equation (5) we have

$$f=j\sin\vartheta-t\cdot\vartheta'+\vartheta''' \tag{7}$$

Thus, the problem of determining the distributed reaction of walls $f$ requires the finding of the axial force $t$. To do this, let us eliminate function $f$ from the system (5)-(6):

$$t'+k_t\vartheta'\cdot t=k_t\vartheta'''-\vartheta'\cdot\vartheta''+k_t j\sin\vartheta-j\cos\vartheta \quad 8)$$

The resulting differential first-order equation is linear, inhomogeneous and, in general, with variable coefficients. We will study and solve the basic differential equation (8).

## Integration of differential equation of axial force

Let us mark the right part (8) in the following way

$$\varphi=k_t\vartheta'''-\vartheta'\cdot\vartheta''+j\left(k_t\sin\vartheta-\cos\vartheta\right) \tag{9}$$

Integration (8) is carried out provided that the coefficient of resistance of the string motion within a single area is constant: $k_t=\text{const}$, $0<k_t<1$. Let us solve the Cauchy problem provided that in the established intersection with the coordinate $s=L$, where $z=Z$, $\vartheta=\Theta$ and axial force $t(L)=t_Z$ is applied.

The general solution of the inhomogeneous equation (8) is sought by the Bernoulli method [5] in the form of a product of two functions: $t = v \cdot w;$ its substitution in (8) gives

$$\begin{gathered} v' \cdot w + v \cdot w' + k_T \vartheta' \cdot v \cdot w = \varphi \\ \left(v' + k_t \vartheta' \cdot v\right) \cdot w + v \cdot w' = \varphi \end{gathered} \tag{10}$$

Since we are looking for one function $t$, then one of the two product functions can be arbitrary. Choose $v$ such that satisfies the homogeneous equation, formed from the expression in brackets (10) and solution of which we can find by separating the variables:

$$v' + k_t \vartheta' \cdot v = 0 \tag{11}$$

$$v = e^{k_T(\Theta - \vartheta)} \tag{12}$$

Now let us substitute expressions (11), (12) and (9) into equation (10) and integrate the resulting differential equation:

$$e^{k_t\Theta}(w - c) = k_t \int_L^s e^{k_t\vartheta} \vartheta''' ds - \int_L^s e^{k_t\vartheta} \vartheta' \vartheta' ds + j \int_L^s e^{k_t\vartheta} (k_t \sin\vartheta - \cos\vartheta) ds \tag{13}$$

where $c$ is constant of integration.

The first of the integrals containing in (13) is found by integrating the parts:

$$\int_L^s e^{k_t\vartheta} \vartheta''' ds = \int_\Theta^\vartheta e^{k_t\vartheta} d(\vartheta'') = e^{k_t\vartheta}\vartheta'' - e^{k_t\Theta}\vartheta''_\Theta - k_t \int_L^s e^{k_t\vartheta} \vartheta' \vartheta'' ds$$

where $\vartheta'_\Theta = \vartheta'(\Theta)$ is the value of the derived function in the intersection, where $\vartheta = \Theta$.

Now the first and second integrals of (13) can be combined:

$$k_t \int_L^s e^{k_t\vartheta} \vartheta''' ds - \int_L^s e^{k_t\vartheta} \vartheta' \vartheta'' ds = k_t\, e^{k_t\Theta} \left(e^{k_t(\vartheta-\Theta)}\vartheta'' - \vartheta''_\Theta\right) - \left(1 + k_t^2\right) \int_L^s e^{k_t\vartheta} \vartheta' \vartheta'' ds$$

The second of integrals (13) is also integrated by parts:

$$\int_L^s e^{k_t\vartheta} \vartheta' \vartheta'' ds = \frac{1}{2} \int_\Theta^\vartheta e^{k_t\vartheta} d\left(\vartheta'^2\right) = \frac{e^{k_t\Theta}}{2} \left(e^{k_t(\vartheta-\Theta)}\vartheta'^2 - \vartheta'^2_\Theta\right) - \frac{k_t}{2} \int_L^s e^{k_t\vartheta} \vartheta'^3 ds$$

Substituting the resulting integrals in (13), we obtain a function $w$:

$$w = k_t\left(e^{k_t(\vartheta-\Theta)}\vartheta'' - \vartheta''_\Theta\right) - \frac{1+k_t^2}{2} \times \left(\left(e^{k_t(\vartheta-\Theta)}\vartheta'^2 - \vartheta'^2_\Theta\right) - \frac{k_t}{e^{k_t\Theta}} \int_L^s e^{k_t\vartheta} \vartheta'^3 ds\right) + \frac{j}{e^{k_t\Theta}} \int_L^s e^{k_t\vartheta} \left(k_t \sin\vartheta - \cos\vartheta\right) ds + c \tag{14}$$

The product of functions (12) and (14) gives a function $t$:

$$t = e^{k_t(\Theta-\vartheta)} w = k_t\left(\vartheta'' - e^{k_t(\Theta-\vartheta)}\vartheta''_\Theta\right) - \frac{1+k_t^2}{2} \times \left(\left(\vartheta'^2 - e^{k_t(\Theta-\vartheta)}\vartheta'^2_\Theta\right) - \frac{k_t}{e^{k_t\vartheta}} \int_L^s e^{k_t\vartheta} \vartheta'^3 ds\right) +$$

$$+ \frac{j}{e^{k_T\vartheta}} \int_L^s e^{k_T\vartheta} \left(k_T \sin\vartheta - \cos\vartheta\right) ds + c\, e^{k_T(\Theta-\vartheta)}$$

Under the conditions of the Cauchy problem we get $c = t_Z$. Consequently, the distribution of the axial force in the body of the column, taking into account the deviation of the well and friction on its walls, has the form

$$t = k_t\left(\vartheta'' - \vartheta''_\Theta e^{k_t(\Theta-\vartheta)}\right) - \frac{1+k_t^2}{2}\times\left(\left(\vartheta'^2 - \vartheta'^2_\Theta e^{k_t(\Theta-\vartheta)}\right) + \frac{k_t}{e^{k_t\vartheta}}\int_s^L e^{k_t\vartheta}\vartheta'^3\,ds\right) - \\ -\frac{j}{e^{k_t\vartheta}}\int_s^L e^{k_t\vartheta}\left(k_t\sin\vartheta - \cos\vartheta\right)ds + t_Z e^{k_t(\Theta-\vartheta)} \qquad (15)$$

In the expression (15) the direction of integration is changed. Transformations helped to get rid of the second and third derivatives under integrals. The last integral (15) can not be simplified in general case. It can be found in quadratures only for the case of a constant radius of the well curvature when $ds = R\,d\vartheta$ [3].

Knowing the axial force $t$ (15), one can find a distributed reaction of the well walls by expression (7).

## Methods of numerical differentiation and integration of inclinometric table

According to the results of directional survey, that is the table of zenith angles $\vartheta$, measured with the interval $\Delta s$, a real well profile is constructed. At first, they determine depth gain $\Delta z$, horizontal displacement $\Delta x$ from the vertical axis of the well in the directional drilling, lateral deviation $\Delta y$ from the directional orientation:

$$\Delta z_n = \Delta s_n \cos\vartheta_n$$

$$\Delta x_n = \Delta s_n \sin\vartheta_n \cos(A_n - Az)$$

$$\Delta y_n = \Delta s_n \sin\vartheta_n \sin(A_n - Az)$$

where:

$n$ – sequence number of measurement;

$\Delta s_n$ – coordinate gain $s$ of intersection along the deviated wellbore, $\Delta s_n = s_n - s_{n-1}$;

$A_n$ – measured magnetic azimuth;

$Az$ – azimuth of directional orientation.

According to calculated gains absolute values of depth $Z_n$, horizontal displacement $X_n$ and lateral deviation $Y_n$ as the sum of gains are determined:

$$Z_n = \sum_{i=1}^{n}\Delta z_i\,, \qquad X_n = \sum_{i=1}^{n}\Delta x_i\,, \qquad Y_n = \sum_{i=1}^{n}\Delta y_i \qquad (16)$$

by which they build a vertical profile and a horizontal well plan.

For a numerical differentiation of a table-defined function, a central scheme is used [6]:

$$d\vartheta_n = \frac{\vartheta_{n+1} - \vartheta_{n-1}}{s_{n+1} - s_{n-1}} \qquad (17)$$

where the letter $d$ denotes numerical differentiation.

According to (17), the values of the second $d2$ and the third $d3$ derivatives can be obtained correspondingly:

$$d2\vartheta_n = \frac{d\vartheta_{n+1} - d\vartheta_{n-1}}{s_{n+1} - s_{n-1}}$$
$$d3\vartheta_n = \frac{d2\vartheta_{n+1} - d2\vartheta_{n-1}}{s_{n+1} - s_{n-1}} \tag{18}$$

Applying the expression (7), the true value of $F_n$ of the distributed response of the well wall in the *n*-th section is found by the formula:

$$F_n = EJf_n = EJj\sin\vartheta_n - T_n \cdot d\vartheta_n + EJ \cdot d3\vartheta_n \tag{19}$$

where $T_n = EJt_n$ is the actual value of the axial force, which must first be found, defining the integrals in the expression (15).

The numerical integration of tabulated functions is carried out according to the trapezoidal rule [6]. For this, the interval of integration $[s, L]$ is divided into elementary intervals; on each of them, they find the area of the trapezoid, constructed on the ordinates of the function at the edges of the interval. The value of an integral is equal to the sum of the squares of all elementary trapezoids.

For an inclinometric table, for an elementary interval, it is natural to choose the measurement interval $\Delta s$, which makes it possible to find the values of the integral functions for the formula (15) at the edges of each interval.

As formula (15) shows, to find the value of the axial force $t$ in the current section $s$, it is necessary to know its value $t_Z$ at the end of the integration interval. The only cross section of the casing, where the axial force is known in advance, is its free end (casing shoe) – here $t_Z = 0$. Proceeding from this, the following method of numerical analysis of inclinometric table is developed.

For the integration interval, choose the measurement interval $\Delta s$. Then in the current section $s_n$, which is the beginning of the interval and where you need to find the axial force $t_n$, one can determine all the values of functions and derivatives necessary for (15). The same values at the end of the interval (when $s = L$ and $\vartheta = \Theta$) are found by the data of the next $(n + 1)$-th measurement.

At the same time, for formula (15) the integral value is equal to the trapezoidal area constructed on the ordinates of the integrands determined according to the *n*-th and $(n + 1)$-th measurements. The value of the trapezoidal area is found as the product of the interval $\Delta s$ to the arithmetic mean of the specified ordinates.

Thus, transforming formula (15) and integrals in it according to the proposed method, the real value of the axial force $T_n$ at each step of integration is determined by the formula

$$\begin{aligned} T_n = \frac{EJ}{e^{k\vartheta_n}}\Bigg[ & k\left(e^{k\vartheta_n} d2\vartheta_n - e^{k\vartheta_{n+1}} d2\vartheta_{n+1}\right) - \frac{1+k^2}{2}\Bigg(e^{k\vartheta_n}\left(d\vartheta_n\right)^2 - e^{k\vartheta_{n+1}}\left(d\vartheta_{n+1}\right)^2 + \\ & + k\left(s_{n+1} - s_n\right)\cdot\frac{e^{k\vartheta_n}\left(d\vartheta_n\right)^3 + e^{k\vartheta_{n+1}}\left(d\vartheta_{n+1}\right)^3}{2}\Bigg)\Bigg] + \\ & + \frac{EJj}{e^{k\vartheta_n}}\cdot\frac{e^{k\vartheta_n}\left(\cos\vartheta_n - k\sin\vartheta_n\right) + e^{k\vartheta_{n+1}}\left(\cos\vartheta_{n+1} - k\sin\vartheta_{n+1}\right)}{2} \times \left(s_{n+1} - s_n\right) + T_{n+1}\frac{e^{k\vartheta_{n+1}}}{e^{k\vartheta_n}} \end{aligned} \tag{20}$$

Beginning with the last *N*-th measurement for which the value $T_N = EJt_Z = 0$ is known, according to (20) we find the previous value $T_{N-1}$, by which we get value $T_{N-2}$ and so on. The determination of the distribution of the axial forces in the body of the pipe occurs from the bottom upwards along the casing from its free end, with the preset value of the axial force for the last measurement.

The design of the casing column is described by setting the diameters $D_n$ of the pipes and the thickness $\delta_n$ of their walls at each depth interval according to the well program; consequently we determine the area $S_n$ of the crosscut of the pipes and its moments of inertia $J_n$.

At each depth interval, we set the mass $m_t$ of one linear meter of the casing pipe, the mass $m_m$ of the collar, the length $l_m$ of the pipes (the distance between the couplings), the mass $m_c$ of the centralizer and the distance $l_c$ between them. Determine the combined mass $m_n$ of the linear meter of the casing column by the formula

$$m_n = m_t + \frac{m_m}{l_m} + \frac{m_c}{l_c} \tag{21}$$

The coefficients of friction are given for each interval of bedding of rocks in accordance with the borehole log. We also set the values of the densities $\gamma_n$ of the drilling fluid, which is in the well after it was washed out before the casing is lowered. The combined weight $j_n$ of the linear casing meter is calculated by the formula

$$j_n = \frac{9.8(\rho - \gamma_n)}{EJ_n} \cdot \frac{m_n}{\rho} \tag{22}$$

where:

$\rho$ – density of the casing material;

$\frac{m_n}{\rho}$ – the volume of its combined cross-section.

Along the wellbore, we find the values of the axial forces and the reactions of the wall by the formulas (20) and (19). The values of the local radii $R_n$ of the curvature and the internal bending moments $M_n$ are calculated by the formulas:

$$R_n = \frac{1}{d\vartheta_n}, \quad M_n = EJq_n = EJd\vartheta_n \tag{23}$$

To determine the strength of the casing, we must determine the local maximal values of internal stresses in the body of the pipes by the sum of stresses from tension and bending:

$$\sigma_{\max} = \frac{T_n}{S_n} + \frac{|M_n|}{J_n}\frac{D_n}{2} = \frac{T_n}{S_n} + \frac{E D_n}{2|R_n|} \tag{24}$$

The value of the bending moment and the radius of curvature is taken modulo to obtain the maximum stress value in the pipe, regardless of the direction of its bend and the location of the stretched fibers.

The developed numerical analysis program has been tested in a test mode by comparing with the results of analytically found formulas of the axial force *t* and reaction of walls *f* for a well section of a constant radius of curvature, taking into account frictional forces [3]. At the same time the error of program calculations was no more than 0.02%.

## Results and Discussion

Approbation of the developed methodology is carried out according to the data of the operating well number 170. However, at first the analysis of its program was carried out using theoretical solutions. For this purpose, in the areas from which the real program is made, the following parameters are calculated according to the formulas obtained analytically in [3]: the distribution of axial forces in the initial and final vertical sections; the distribution of axial forces and the reactions of the walls on the radius of zenith angle buildup, on two inclined rectilinear sections and two radius sections of the zenith angle decline. The values of the radii of curvature and bending moments are calculated according to the formulas (23), the maximum stresses – according to (24). The results of calculations are given in table 1.

**Table 1.** *Theoretical characteristics of the well program No. 170*

| Casing length intervals, m | Characte-ristics of areas | Diameter *D* of the string, thickness $\delta$ of pipe wall, mm | Zenith angle $\vartheta$, angle gain $\Delta\vartheta$ for 10 m | Reaction $F = EJf$ of well wall, kN/m | Radius *R* of curvature, m | Bending moment $M = EJq$, kN/m | Tension jump in the pipe body, MPa |
|---|---|---|---|---|---|---|---|
| 0…1200 | | 168×10.6 | 0° | 0 | – | 0 | – |
| 1200…1350 | vertical | | 0° | 0 | – | 0 | +29.2 |
| 1350…1650 | Zenith angle buildup | | $\vartheta = 0…14°.5$<br>$\Delta\vartheta = 0°.5$ | −0.85 …<br>… −0.66 | +1146 | +1.83 | +12.7<br>−12.7 |
| 1650…2400 | inclined | 146×10.7 | $\vartheta = 14°.5$ | +0.077 | – | 0 | – |
| 2400…2480 | Zenith angle decline | | $\vartheta = 14°.5…13°.7$<br>$\Delta\vartheta = -0°.1$ | +0.19 …<br>… +0.18 | −5730 | −0.37 | +2.6<br>−2.6 |
| 2480…3700 | inclined | | | +0.073 | | | – |
| 3700…3800 | | | $\vartheta = 13°.7$ | +0.068 | – | 0 | +3.3 |
| 3800…4150 | Zenith angle decline | 140×10.5 | $\vartheta = 13°.7…0°$<br>$\Delta\vartheta = -0°.4$ | +0.23 …<br>… +0.11 | −1432 | −1.25 | +9.8<br>−9.8 |
| 4150…4680 | vertical | | 0° | 0 | – | 0 | – |

The theoretical analysis of the well design has shown that the axial force in the body of the column with increasing depth decreases piecewise linearly on straight sections (vertical and inclined), as well as on the curved ones along a circular arc. The latter is due to the small values of the zenith angles, which is consistent with the results of [3]. The same nature has the distribution of tensile stresses in the body of the pipes. At the same time, a discontinuous change in the stresses of two types was detected. The first type (on the marks of 1200 m and 3700 m) is caused by a change in the standard size of the casing pipes.

The second type of stress jumping is typical for the column curving intervals (with a constant radius of curvature according to the program) and is determined by the value of the bending moment created by the curvature. In addition, jumps of well wall reactions in the regions of the conjugation of its rectilinear and curved areas occur.

The jump-like nature of the change in the stresses and reactions of the walls is due to the fact that in the transition from rectilinear areas of the design well to those curved along a circular arc there is no geometric break of its axis, since the tangents coincide in the transitional section. However, the jump in the bending moment occurs, which is on the arc and is proportional to the curvature, but is absent on a straight line.

This is a consequence of the idealization of the project, in the first place, through the description of the distorted areas by the arc of the ideal circle. In a real well, the diameter of which is slightly larger than the diameter of the pipes, the edges of the column at the conjugated sites due to the elasticity of the pipes receive variable curvatures, which acquire values from $R^{-1}$ on the arc section to 0 on a straight line and vice versa.

A positive well reaction indicates that the casing column rests on its lower wall; this is observed on inclined rectilinear and in the areas of the decline of the zenith angle (table 1). The negative reaction of the walls shows that the column rests on the upper wall of the well due to the forces of elasticity of the initially rectilinear casing; this is manifested in the area of the zenith angle buildup. On the inclined sections, the reaction of the well coincides with the reaction of the inclined plane. On curved areas, the reaction of the walls varies, but given the small lengths of the arcs, its change can be considered linear. These results are consistent with the conclusions [3].

The numerical analysis program also worked out the program of the well number 170, given in the form of inclinometric table; the results of this are presented in figure 2 by lines *1*. Patterns of the distribution of axial forces, the reactions of the well walls, bending moments, maximum stresses in the body of the column, obtained by numerical analysis and calculated by analytical formulas [3], qualitatively coincide completely.

The quantitative evaluation showed that the greatest difference between the numerical and theoretical calculation of axial forces in the project is observed near the conjugation of rectilinear and arc sections of the well. In the intersection between the vertical section and the arc with the zenith angle buildup it reaches 3.2%, in the intersection between the inclined and the arc with a downward angle it reaches 3.0%. On average, on straight and long arc sections, the difference is 1.5...2.5%.

The difference between the numerical and theoretical calculation of the reactions of the walls of the well project is found only in the arc sections (an average of 0.7...1.5%). The greatest difference is near the jump of the reaction value: 2.8% at the beginning of the section of zenith angle buildup and 2.1% at the beginning of the decline. On rectilinear sections (in particular, on inclined ones), the calculations of reactions give the same values.

The error of the developed numerical analysis method is due to the inaccuracy of numerical differentiation and integration and depends first of all on the choice of the value of the interval [6]. The difference between the numerical and theoretical calculations of the design maximum stresses in the body of the pipes is 0.01...0.03% along the entire column.

In addition, the developed program of numerical analysis worked out inclinometric table of data of field measurements of the actual well number 170; the results of this are presented in figure 2 by lines *2*. This allowed to reveal the following features of the behavior of the casing in a real drilled borehole.

The graphs of the theoretical and actual axial forces practically coincide (fig. 2b). The difference between them on the vertical section increases from 2.2% to 3.3% in the cross section where the bending of the well and the zenith angle buildup begin. The greatest value of this difference lies in the lower part of the inclined area and in the transition from the inclined to the vertical one – to 4.6%. On other inclined and deviated areas the difference is 1...3%. However, these estimates of axial forces can not be attributed to the error of the numerical method (since different data have been processed – design and actually measured ones). First, they indicate a satisfactory coincidence in the whole of the design and drilled wells, as shown in figure 2a.

The actual deviation of the real well profile from the design one is shown by a graph of bending moments (fig. 2d), which can also be considered as a graph for changing the actual curvature of the well, since they are proportional according to (23). As you can see, the axis of an actual drilled well significantly deviates from the design profile (rectilinear or radius one). This is evidenced by the continuous change in bending moments by both magnitude and direction, which is caused by a change in the actual values of the local curvature of the well. This is due to the impact of a large number of technical, technological and geological factors on the drilling process.

Under these conditions, a casing column, trying to preserve its initially rectilinear form, at the expense of the forces of elasticity rests on opposite walls of a stochastically curved well, causing variables in magnitude and direction of reaction (fig. 2c). By comparing figures 2c and 2d, we can see that the magnitude and change of the local curvature of the well causes a proportional value and a change in the reaction of its wall. The reaction of the wall is also proportional to the bending rigidity of the casing. Accordingly, the internal bending moment and bending stress in the body of the pipe also change.

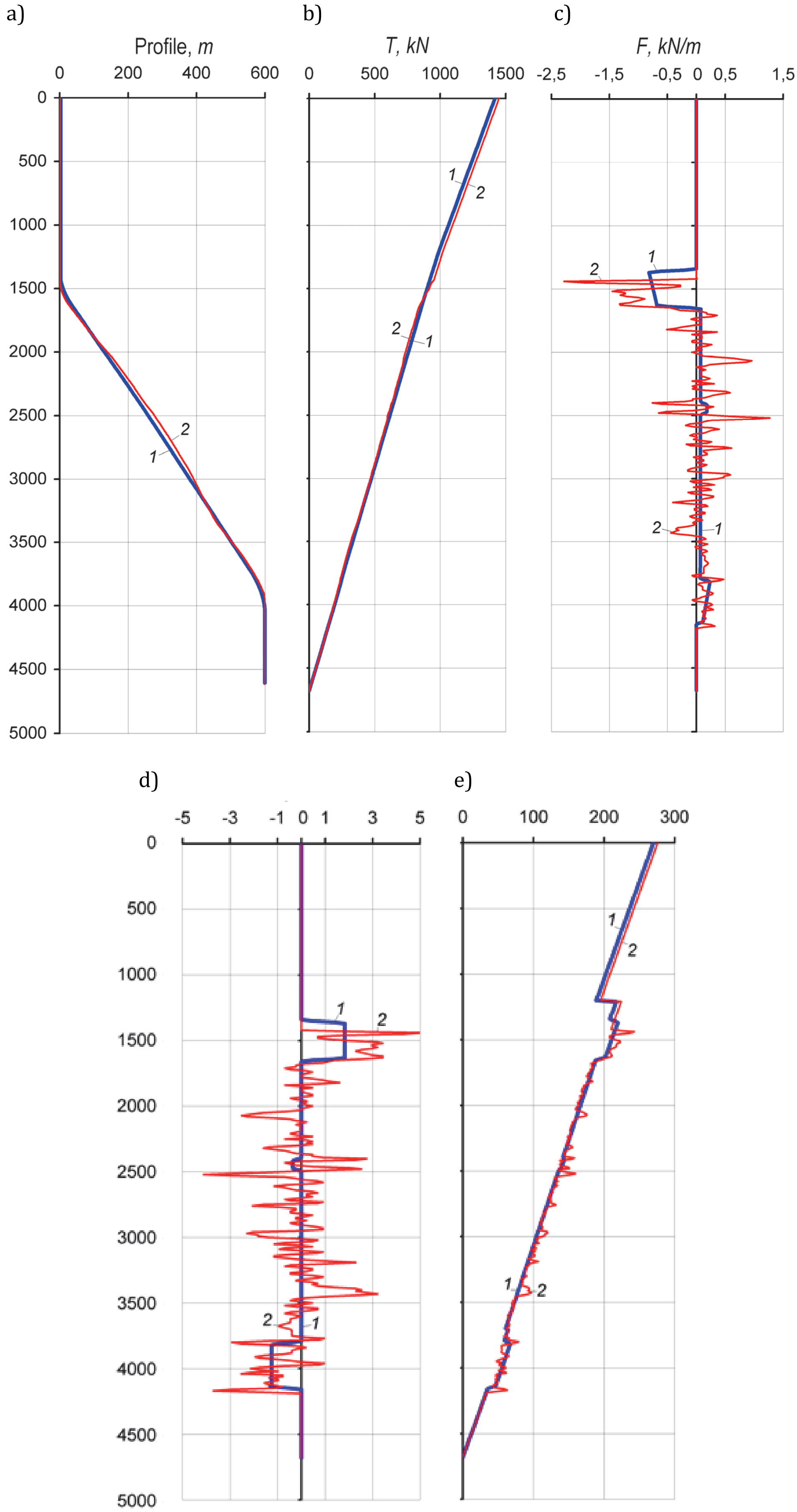


***Figure 2.*** *Wells profile (a), graphs of axial forces T (b), walls reactions F (c), bending moments M (d) and normal tensions σ (e) in column combined at depth intervals; where: 1 – according to the well project, 2 – according to inclinometric measurements*

The largest jump in the values of the reaction of the actual well walls, the bending moment and the maximum stresses in the body of the casing is observed at a mark of 1440 m, where the actual deviation of the well and zenith angle buildup (as opposed to the design of 1350 m) begin. Along with this, the results of numerical analysis of an actual well made it possible to detect its areas with a significant increase in the curvature and the reaction of the wall. These are areas where the forced deviation of the well (zenith angle buildup and decline) occurred. In addition, in the areas of stabilization of the zenith angle, one can also find a local increase in the curvature and the reaction of the wall.

Numerical analysis of the stresses shows that for this well profile the tensile stress of the column is dominant (fig. 2e). Local stresses are of fluctuating nature and are related to the increase of local curvature of the well and bending moment in the column.

## Applying results

The developed method of numerical analysis of the well allows detecting the areas with a significant local increase in the curvature, which indicates their obstructed passability. It allows one to accurately determine the depth intervals for increasing the well diameter. This must be done before lowering the casing string.

In addition, according to the results of the analysis, it is possible to determine the parameters of the stress-strain state of the casing, which can be used to predict its working capacity and operating life in the curved well.

## Conclusions

The stress-strain state of the casing in the curved well can be determined by the non-uniform system of differential equations, which describes the bending of a long elastic rod under the action of distributed forces of its own weight, the reaction of supports and friction. Having the shape of the well with a known function of the zenith angle, we can find the solutions of the system in the form of functions of the distribution of axial forces and bending moments in the body of the column, as well as the reactions of the walls, which lead the column to the actual well profile.

Parameters of the stress-strain state of the casing in an actually drilled well can be determined by the developed methods of numerical integration of the data of inclinometric measurements of the well and the software of their numerical analysis. This allows us to identify areas of local increase in curvature and obstructed passability of the curved well.

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